Logical Theory of Evaluative Linguistic Expressions II

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Outline

What we are speaking about?

- Evaluative linguistic expressions:
  small, medium, big, twenty five, roughly one hundred, very short, more or less strong, not tall, about twenty five, the sea is deep but not very, roughly small or medium, very roughly strong
- Evaluative linguistic predications:
  weight is small, pressure is very high, extremely rich person

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Fuzzy type theory — syntax

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- R. Carnap, K. Gödel, A. Tarski, A. Turing, A. Church, L. Henkin, P. Andrews, P. Martin-Löf

Types

Elementary types: o (truth values), ϵ (objects)
Composed types: $\beta\alpha$

Each formula has a certain type: $A_\alpha \in Form_\alpha$
Fuzzy equality: $A_\beta \equiv B_\beta$
$\lambda x_\alpha\, C_\beta$ — formula of type $\beta\alpha$ (*lambda term*)
Delta connective: Δ_{oo} (*surely*)
Description operator $\iota_{\alpha(o\alpha)}$
Formulas of type o are propositions

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Logical connectives

- **Equivalence:** $A_o \equiv B_o$ **basic connective!**
- **Implication:** $A_o \Rightarrow B_o$
- **Disjunction:** $A_o \vee B_o$
- **Conjunction:** $A_o \wedge B_o$, interpreted by minimum ($\wedge$)
  phrasal conjunction
- **Strong conjunction:** $A_o \,\&\, B_o$, interpreted by $\otimes$
  sentential conjunction
- **Delta connective (surely):** ΔA_o

Local character of conjunction: resulting truth degree depends on the meaning of conjuncts
(*slow and safe car; big and beautiful house*)

Modus ponens requires strong conjunction

$$\frac{a/A_o,\; c/A_o \Rightarrow B_o}{a \otimes c/B_o}$$

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Structure of truth values

Łukasiewicz Δ-algebra ($\mathcal{L}_\Delta$) (IMTL$_\Delta$-algebra)

$$\mathcal{L} = \langle [0,1], \vee, \wedge, \otimes, \Delta, \rightarrow, 0, 1 \rangle$$

$\vee, \wedge$ minimum, maximum

$$a \otimes b = 0 \vee (a + b - 1), \qquad (\otimes = \text{left continuous t-norm})$$

$$a \rightarrow b = 1 \wedge (1 - a + b) \qquad (\rightarrow = \text{residuation})$$

$$\neg a = a \rightarrow 0 = 1 - a, \qquad \neg\neg a = a$$

$$\Delta(a) = \begin{cases} 1 & \text{if } a = 1, \\ 0 & \text{otherwise.} \end{cases} \qquad a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a)$$

Fuzzy equality

Fuzzy relation $=_\alpha$: $M_\alpha \times M_\alpha \longrightarrow L$

$$[x =_\alpha x] = 1 \qquad \text{(reflexivity)}$$

$$[x =_\alpha y] = [y =_\alpha x] \qquad \text{(symmetry)}$$

$$[x =_\alpha y] \otimes [y =_\alpha z] \leq [x =_\alpha z] \qquad \text{(transitivity)}$$

Example

$$[x =_\alpha y] = 0 \vee (1 - |x - y|)$$

$$[x =_\alpha y] = \begin{cases} 1, \text{if } x = y \\ \frac{1}{v-u} \cdot ((v - x) \wedge (v - y)) \vee ((x - u) \wedge (y - u)) \end{cases}$$

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Semantics – frame

Frame $\mathcal{M}$

$$\mathcal{I} = \langle (M_\alpha, =_\alpha)_{\alpha \in Types}, \mathcal{L}_\Delta \rangle$$

$$(M_o = \{a \mid a \in L\}, \leftrightarrow) \qquad (M_\epsilon = \{u \mid \varphi(u)\}, =_\epsilon)$$

$$(M_{oo} \subseteq \{g_{oo} \mid g_{oo} : M_o \longrightarrow M_o\}, =_{oo})$$

$$(M_{o\epsilon} \subseteq \{f_{o\epsilon} \mid f_{o\epsilon} : M_\epsilon \longrightarrow M_o\}, =_{o\epsilon})$$

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Interpretation

$$\mathcal{I}^{\mathcal{M}}(A_{\beta\alpha}) = f_{\beta\alpha} \in M_{\beta\alpha} \subseteq M_{\beta}^{M_{\alpha}}$$

Interpretation of formulas $A_{\beta\alpha}$: functions $M_{\alpha} \longrightarrow M_{\beta}$

Example

- $\mathcal{I}^{\mathcal{M}}(A_o) \in L$ – a truth value
- $\mathcal{I}^{\mathcal{M}}(A_{o\epsilon})$ — fuzzy set in M_{ϵ}
- $\mathcal{I}^{\mathcal{M}}(A_{(o\beta)\alpha})$ — fuzzy relation in $M_{\alpha} \times M_{\beta}$
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- $\mathcal{I}^{\mathcal{M}}(\Delta A_o) \in \{0, 1\}$ – crisp truth value

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Logical axioms

17 axioms

- Equality axioms
  (FT1) $\mathbf{\Delta}(x_\alpha \equiv y_\alpha) \Rightarrow (f_{\beta\alpha}\, x_\alpha \equiv f_{\beta\alpha}\, y_\alpha)$
- Truth values axioms
  (FT6) $(x_o \equiv y_o) \equiv ((x_o \Rightarrow y_o) \wedge (y_o \Rightarrow x_o))$
- Delta axioms
  (FT5) $(g_{oo}(\mathbf{\Delta} x_o) \wedge g_{oo}(\neg\mathbf{\Delta} x_o)) \equiv (\forall y_o) g_{oo}(\mathbf{\Delta} y_o)$
- Predicate axiom
  (FT16) $(\forall x_\alpha)(A_o \Rightarrow B_o) \Rightarrow (A_o \Rightarrow (\forall x_\alpha) B_o)$
  x_α is not free in A_o
- Axiom of descriptions
  (FT17) $\iota_{\epsilon(o\epsilon)}(\mathbf{E}_{(o\epsilon)\epsilon}\, y_\epsilon) \equiv y_\epsilon$

Inference rules and provability

Rule (R)

Let $A_\alpha \equiv A'_\alpha$ *and* $B \in Form_o$*. Then we infer* B' *where* B' *comes from* B *by replacing one occurrence of* A_α*, which is not preceded by* λ*, by* A'_α*.*

Rule (N)

Let $A_o \in Form_o$ *be a formula. Then from* A_o *infer* ΔA_o*.*

Formal theory T of FTT is a set of formulas of type o

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Completeness

Theorem

(a) *A theory T of fuzzy type theory is consistent iff it has a general model $\mathcal{M}$.*

(b) *For every theory T of the fuzzy type theory and a formula A_o*

$$T \vdash A_o \quad \text{iff} \quad T \models A_o.$$

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Formal theory of TEv-expressions

We construct a formal theory T^{Ev} in the language of FTT formalizing general 6 characteristics

All properties are consequences of 11 special axioms of T^{Ev}

Formal syntactical proofs of all properties!

IRAFM

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Axioms of T^{Ev}

(EV1) $(\exists z)\Delta(\neg z \equiv z)$

(EV2) $(\bot \equiv w^{-1}\bot_w) \wedge (\dagger \equiv w^{-1}\dagger_w) \wedge (\top \equiv w^{-1}\top_w)$

(EV3) $t \sim t$

(EV4) $t \sim u \equiv u \sim t$

(EV5) $t \sim u \,\&\, u \sim z \cdot \Rightarrow t \sim z$

(EV6) $\neg(\bot \sim \dagger)$

(EV7) $\Delta((t \Rightarrow u)\&(u \Rightarrow z)) \Rightarrow \cdot t \sim z \Rightarrow t \sim u$

(EV8) $t \equiv t' \,\&\, z \equiv z' \Rightarrow \cdot t \sim z \Rightarrow t' \sim z'$

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(EV10) $\mathit{NatHedge}\,\bar{\nu}\,\&(\exists \nu)(\exists \nu')(\mathit{Hedge}\,\nu\,\&\,\mathit{Hedge}\,\nu'\,\&$ $(\nu_1 \preceq \bar{\nu} \wedge \bar{\nu} \preceq \nu_2))$

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IRAFM

Context

(A) Nonempty, linearly ordered and bounded scale, three distinguished limit points: *left bound*, *right bound*, and a *central point*

Context $w_{\alpha o}$ $\mathcal{I}(w_{\alpha o}) = w : [0,1] \longrightarrow M$:

$$w(0) = v_L \qquad \text{(left bound)}$$
$$w(0.5) = v_S \qquad \text{(central point)}$$
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Set of contexts $W = \{w \mid w : [0,1] \longrightarrow M\}$
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IRAFM

Intension

(B) Function from the set of contexts into a set of fuzzy sets

$\mathrm{Int}(\mathcal{A}) = \lambda w\, \lambda x\, (Aw)x \qquad \mathcal{I}(\mathrm{Int}(\mathcal{A})) : W \longrightarrow \mathcal{F}(w([0,1]))$

Scheme of intension

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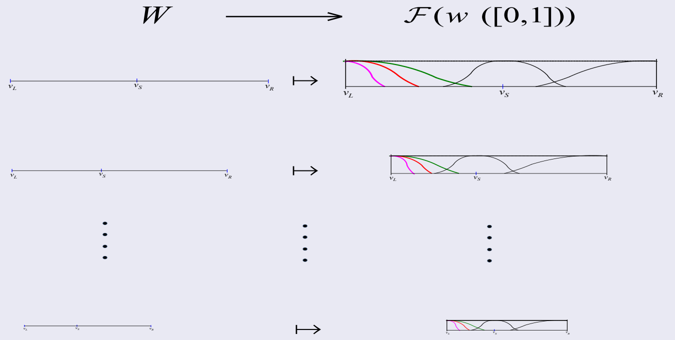

Horizon

(C) Each of the limit points is a starting point of some *horizon* running from it in the sense of the ordering of the scale towards the next limit point (the horizon vanishes beyond)

Three horizons

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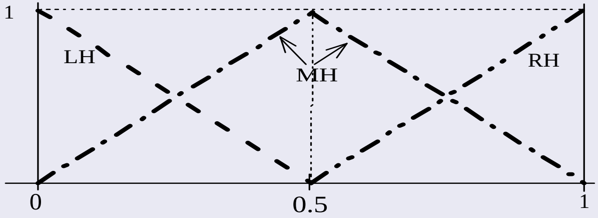

Special fuzzy equality

We introduce a special fuzzy equality $\sim$ on the set $[0, 1]$ of truth values

Example

Standard Łukasiewicz MV-algebra of truth values:

$$[a \sim b] = \frac{0.5 - |a - b|}{0.5}.$$

Fuzzy equality induced by $\sim$

$$y \approx_w y' := \quad \text{iff} \quad w^{-1}y \sim w^{-1}y', \quad x, y \in w$$

Three horizons

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$$
\begin{aligned}
LH(a) &= [0 \sim a], & LH(w\,x) &= [v_L \approx_w x] \\
MH(a) &= [0.5 \sim a], & MH(w\,x) &= [v_S \approx_w x] \\
RH(a) &= [1 \sim a], & RH(w\,x) &= [v_R \approx_w x]
\end{aligned}
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Properties of horizon

(D) Each horizon is represented by a special fuzzy set determined by a reasoning analogous to that leading to the sorites paradox.

Sorites paradox

One grain does not make a heap. Adding one grain to what is not yet a heap does not make a heap. Consequently, there are no heaps.

Falakros paradox

A man with no hair is bald. A man with one hair more than a bald man is still bald. Consequently, all men are bald.

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Sorites in fuzzy logic

New predicate $\mathbb{FN}(n)$: *"n is small"; "n is feasible"; "n is finite"*

Axioms

- *"there is a small number"* — valid
- *"if n is small then $n + 1$ is also small"* — practically valid!
- *"there is a number not being small"* — valid

No contradiction!
(impossible in classical logic)

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Sorites in the theory of TEv-expression

Special context w_N:

$$v_L = 0, \quad v_S = p, \quad v_R = q$$

$\approx_{w_N}$ – induced fuzzy equality in the context w_N

$$\mathbb{FN} := \lambda n \cdot \mathbf{0} \approx_{w_N} n$$

Finite numbers do not form a heap

Interpretation: fuzzy set $\mathbb{FN}(n) = [0 \approx_{w_N} n]$

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(b) $\vdash n \in w_N \,\&\, \Delta(p \leq n) \Rightarrow \neg\mathbb{FN}\, n$

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(d) $\vdash \neg(\exists n)(n \in w_N \,\&\, \Delta\mathbb{FN} n \,\&\, \Delta\neg\mathbb{FN}(n+1))$

(e) $\vdash n \in w_N \Rightarrow (\mathbb{FN} n \Rightarrow \cdot(n \approx_{w_N} n+1) \Rightarrow \mathbb{FN}(n+1))$

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IRAFM

Hedge: shift of the horizon

(E) *Extension* of each TEv-expression is delineated by *modification (shifting) of the horizon*
Modification: linguistic hedge

Hedges — horizon modifications

$$\nu : [0, 1] \longrightarrow [0, 1]$$

IRAFM

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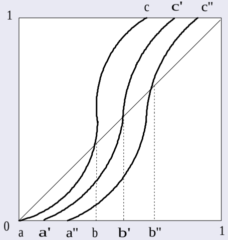
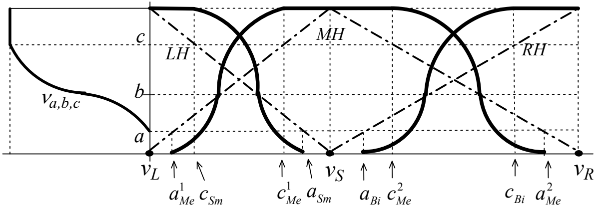


Fundamental evaluative trichotomy

(F) Each scale is vaguely partitioned by the fundamental evaluative trichotomy

Special hedge $\bar{\nu}$



Intensions of evaluative predications

⟨linguistic hedge⟩ $\mapsto \boldsymbol{\nu}$

- Int(X is ⟨linguistic hedge⟩ small) $:=$
$$Sm\,\nu = \lambda w\,\lambda x \cdot \boldsymbol{\nu}(LH(w^{-1}x))$$
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$$(Ev\,\nu)w \subsetsim w([0,1])$$

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Intensions of "small" and "big" have in each context the basic property of antonyms

Theorem

For all w and $x \in w$

$$\vdash \Delta\neg((\overline{Sm}\, w)x \wedge (\overline{Bi}\, w)x)$$

Each context is fully covered by the fundamental evaluative trichotomy

Theorem

For all w and $x \in w$

$\vdash (Eval\ w\ x\ \overline{Sm}) \vee (Eval\ w\ x\ \overline{Me}) \vee (Eval\ w\ x\ \overline{Sm})$

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Vagueness of “small” and “big”

Theorem

No last small natural number

$\vdash \neg(\exists n)(n \in w_N \,\&\, \Delta(Sm\,\nu)w_N n \,\&\, \Delta\neg(Sm\,\nu)w_N(n+1))$

Theorem

- *There is no last small value*
  $\vdash \neg(\exists x)(\forall y)(\Delta(Sm\,\nu)wx \,\&(x <_w y \Rightarrow \Delta\neg(Sm\,\nu)wy))$
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Theorem

No last small natural number

$\vdash \neg(\exists n)(n \in w_N \,\&\, \Delta(Sm\,\nu)w_N n \,\&\, \Delta\neg(Sm\,\nu)w_N(n+1))$

Theorem

- *There is no last small value*

  $\vdash \neg(\exists x)(\forall y)(\Delta(Sm\,\nu)wx \,\&\, (x <_w y \Rightarrow \Delta\neg(Sm\,\nu)wy))$
- *There is no first big value*

  $\vdash \neg(\exists x)(\forall y)(\Delta(Bi\,\nu)wx \,\&\, (y <_w x \Rightarrow \Delta\neg(Bi\,\nu)wy))$

Theorem

Formal theory T^{Ev} of evaluative linguistic expressions is consistent